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# Lepton Polarization in the Decays

$$B \rightarrow X_s \mu^+ \mu^- \text{ and } B \rightarrow X_s \tau^+ \tau^-$$

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## ABSTRACT

The effective Hamiltonian for the decay  $b \rightarrow s l^+ l^-$  predicts a characteristic polarization for the final state lepton, which can serve as an important test of the underlying theory. The lepton polarization has, in addition to a longitudinal component  $P_L$ , two orthogonal components  $P_T$  and  $P_N$ , lying in and perpendicular to the decay plane which are proportional to  $m_l/m_b$ , and therefore significant for the  $\tau^+ \tau^-$  channel. The normal polarization component  $P_N$  is a  $T$ -odd effect connected with the nonhermiticity of the effective Hamiltonian, arising mainly from  $c\bar{c}$  intermediate states. We calculate all three polarization components for the decay  $B \rightarrow X_s \tau^+ \tau^-$  as a function of the lepton pair mass, and find average values  $\langle P_L \rangle_\tau = -0.37$ ,  $\langle P_T \rangle_\tau = -0.63$ ,  $\langle P_N \rangle_\tau = 0.03$ . By comparison, the  $\mu^-$  polarization is  $\langle P_L \rangle_\mu = -0.77$ ,  $\langle P_T \rangle_\mu = \langle P_N \rangle_\mu \approx 0$ .

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## I. INTRODUCTION

The decay  $B \rightarrow X_s l^+ l^-$  has received considerable attention as a potential testing ground for the effective Hamiltonian describing flavour-changing neutral current processes in  $B$  decay (see e.g. Ref. [1]). This Hamiltonian contains the one-loop effects of the electroweak interaction, which are sensitive to the top-quark mass [2–4]. In addition, there are important QCD corrections [5–8], which have recently been calculated in next-to-leading order [4, 9]. The inclusive distributions have been studied in [5, 10, 11], while the exclusive channels  $B \rightarrow Kl^+l^-$  and  $B \rightarrow K^*l^+l^-$  have been analysed in [3, 12]. Recently, attention has been drawn to the fact that the longitudinal polarization of the lepton,  $P_L$ , in  $B \rightarrow X_s l^+l^-$  is an important observable, that may be especially accessible in the mode  $B \rightarrow X_s \tau^+\tau^-$  [13]. The purpose of this paper is to show that complementary information is contained in the two orthogonal components of polarization ( $P_T$ , the component in the decay plane, and  $P_N$ , the component normal to the decay plane), both of which are proportional to  $m_l/m_b$ , and therefore significant for the  $\tau^+\tau^-$  channel. The normal component  $P_N$ , in particular, is a novel feature, since it is a  $T$ -odd observable, that comes about because of the nonhermiticity of the effective Hamiltonian, associated with real  $c\bar{c}$  intermediate states.

## II. SHORT-DISTANCE CONTRIBUTIONS

The effective short-distance Hamiltonian for  $b \rightarrow s l^+ l^-$  [4, 5, 7, 8] leads to the QCD-corrected matrix element

$$\begin{aligned} \mathcal{M} = & \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} \left\{ c_9^{\text{eff}} (\bar{s} \gamma_\mu P_L b) \bar{l} \gamma^\mu l + c_{10} (\bar{s} \gamma_\mu P_L b) \bar{l} \gamma^\mu \gamma^5 l \right. \\ & \left. - 2 c_7^{\text{eff}} \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_b P_R + m_s P_L) b \bar{l} \gamma^\mu l \right\}, \end{aligned} \quad (2.1)$$

where  $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ , and the analytic expressions for the Wilson coefficients are given in Ref. [4]. We use in our analysis the parameters given in the Appendix and

obtain in leading logarithmic approximation

$$c_7^{\text{eff}} = -0.315, \quad c_{10} = -4.642, \quad (2.2)$$

and in next-to-leading order

$$\begin{aligned} c_9^{\text{eff}} = & c_9 \tilde{\eta}(\hat{s}) + g(\hat{m}_c, \hat{s}) (3c_1 + c_2 + 3c_3 + c_4 + 3c_5 + c_6) - \frac{1}{2} g(\hat{m}_s, \hat{s}) (c_3 + 3c_4) \\ & - \frac{1}{2} g(\hat{m}_b, \hat{s}) (4c_3 + 4c_4 + 3c_5 + c_6) + \frac{2}{9} (3c_3 + c_4 + 3c_5 + c_6), \end{aligned} \quad (2.3)$$

with

$$\begin{aligned} (3c_1 + c_2 + 3c_3 + c_4 + 3c_5 + c_6) &= 0.359, \\ (4c_3 + 4c_4 + 3c_5 + c_6) &= -6.749 \times 10^{-2}, \\ (3c_3 + c_4 + 3c_5 + c_6) &= -1.558 \times 10^{-3}, \\ (c_3 + 3c_4) &= -6.594 \times 10^{-2}, \\ c_9 \tilde{\eta}(\hat{s}) &= 4.227 + 0.124 \omega(\hat{s}), \end{aligned} \quad (2.4)$$

where we have introduced the notation  $\hat{s} = q^2/m_b^2$ ,  $\hat{m}_i = m_i/m_b$ . The function  $\omega(\hat{s})$  represents the one-gluon correction<sup>1</sup> to the matrix element of the operator  $\mathcal{O}_9$ , while  $g(\hat{m}_i, \hat{s})$  arises from the one-loop contributions of the four-quark operators  $\mathcal{O}_1$  and  $\mathcal{O}_2$ , and is given by

$$\begin{aligned} g(\hat{m}_i, \hat{s}) = & -\frac{8}{9} \ln(\hat{m}_i) + \frac{8}{27} + \frac{4}{9} y_i - \frac{2}{9} (2 + y_i) \sqrt{|1 - y_i|} \\ & \times \left\{ \Theta(1 - y_i) \left( \ln \left( \frac{1 + \sqrt{1 - y_i}}{1 - \sqrt{1 - y_i}} \right) - i\pi \right) + \Theta(y_i - 1) 2 \arctan \frac{1}{\sqrt{y_i - 1}} \right\}, \end{aligned} \quad (2.5)$$

where  $y_i \equiv 4\hat{m}_i^2/\hat{s}$ .

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<sup>1</sup>See Refs. [4] and [9]. Here we neglect corrections due to a nonzero lepton mass. This will be discussed in a further publication.

### III. LONG-DISTANCE CONTRIBUTIONS

In addition to the short-distance interaction defined by Eqs. (2.1)–(2.4) it is possible to take into account long-distance effects, associated with real  $c\bar{c}$  resonances in the intermediate states, i.e. with the reaction chain  $B \rightarrow X_s + V(c\bar{c}) \rightarrow X_s l^+ l^-$ . This can be accomplished in an approximate manner through the substitution [14]

$$g(\hat{m}_c, \hat{s}) \longrightarrow g(\hat{m}_c, \hat{s}) - \frac{3\pi}{\alpha^2} \sum_{V=J/\psi, \psi', \dots} \frac{\hat{m}_V \text{Br}(V \rightarrow l^+ l^-) \hat{\Gamma}_{\text{total}}^V}{\hat{s} - \hat{m}_V^2 + i \hat{m}_V \hat{\Gamma}_{\text{total}}^V}, \quad (3.1)$$

where the properties of the vector mesons are summarized in Table I. There are six

TABLE I: Charmonium ( $c\bar{c}$ ) masses and widths [15].

Meson	Mass (GeV)	Br( $V \rightarrow l^+ l^-$ )	$\Gamma_{\text{total}}$ (MeV)
$J/\Psi(1S)$	3.097	$6.0 \times 10^{-2}$	0.088
$\Psi(2S)$	3.686	$8.3 \times 10^{-3}$	0.277
$\Psi(3770)$	3.770	$1.1 \times 10^{-5}$	23.6
$\Psi(4040)$	4.040	$1.4 \times 10^{-5}$	52
$\Psi(4160)$	4.159	$1.0 \times 10^{-5}$	78
$\Psi(4415)$	4.415	$1.1 \times 10^{-5}$	43

known resonances in the  $c\bar{c}$  system that can contribute to the decay modes  $B \rightarrow X_s e^+ e^-$  and  $B \rightarrow X_s \mu^+ \mu^-$ . Of these, all except the lowest  $J/\psi(3097)$  contribute to the channel  $B \rightarrow X_s \tau^+ \tau^-$ , for which the invariant mass of the lepton pair is  $s > 4m_\tau^2$ .<sup>2</sup>

<sup>2</sup>As noted by several authors [10, 16], the ansatz (3.1) for the resonance contribution implies an inclusive direct  $J/\psi$  production rate  $\text{Br}(B \rightarrow J/\psi X_s) = 0.15\%$  that is  $\sim 5$  times smaller than the measured  $J/\psi$  rate [17]. This is usually amended by the introduction of a phenomenological factor  $\kappa_V \approx 2$  multiplying the Breit-Wigner function in (3.1). In the present paper, the only observable that is significantly affected by this change is the polarization component  $P_N$ , where we will show the results for both  $\kappa_V = 1$  and  $\kappa_V = 2.35$ .

Alternatively, we can express  $g(\hat{m}_i, \hat{s})$ , Eq. (2.5), through the renormalized vacuum polarization  $\Pi_{\text{had}}^\gamma(\hat{s})$ , which is related to the experimentally measurable quantity  $R_{\text{had}}(\hat{s}) \equiv \sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  via a dispersion relation [18], i.e.

$$\text{Re } \Pi_{\text{had}}^\gamma(\hat{s}) = \frac{\alpha \hat{s}}{3\pi} P \int_{4\hat{m}_\pi^2}^{\infty} \frac{R_{\text{had}}(\hat{s}')}{\hat{s}'(\hat{s}' - \hat{s})} d\hat{s}', \quad \text{and} \quad \text{Im } \Pi_{\text{had}}^\gamma(\hat{s}) = \frac{\alpha}{3} R_{\text{had}}(\hat{s}) , \quad (3.2)$$

where  $P$  denotes the principal value. Using these relations, one finds for the  $c\bar{c}$  contribution

$$\text{Im } g(\hat{m}_c, \hat{s}) = \frac{\pi}{3} R_{\text{had}}^{c\bar{c}}(\hat{s}) , \quad (3.3)$$

and

$$\text{Re } g(\hat{m}_c, \hat{s}) = -\frac{8}{9} \ln \hat{m}_c - \frac{4}{9} + \frac{\hat{s}}{3} P \int_{4\hat{m}_D^2}^{\infty} \frac{R_{\text{had}}^{c\bar{c}}(\hat{s}')}{\hat{s}'(\hat{s}' - \hat{s})} d\hat{s}' . \quad (3.4)$$

The cross-section ratio  $R_{\text{had}}^{c\bar{c}}$  may be written as

$$R_{\text{had}}^{c\bar{c}}(\hat{s}) = R_{\text{cont}}^{c\bar{c}}(\hat{s}) + R_{\text{res}}^{c\bar{c}}(\hat{s}) , \quad (3.5)$$

where  $R_{\text{cont}}^{c\bar{c}}$  and  $R_{\text{res}}^{c\bar{c}}$  denote the contributions from the continuum and the narrow resonances, respectively. The latter is given by the Breit-Wigner formula

$$R_{\text{res}}^{c\bar{c}}(\hat{s}) = \sum_{V=J/\psi, \psi', \dots} \frac{9\hat{s}}{\alpha^2} \frac{\text{Br}(V \rightarrow l^+l^-) \hat{\Gamma}_{\text{total}}^V \hat{\Gamma}_{\text{had}}^V}{(\hat{s} - \hat{m}_V^2)^2 + \hat{m}_V^2 \hat{\Gamma}_{\text{total}}^{V^2}} , \quad (3.6)$$

whereas  $R_{\text{cont}}^{c\bar{c}}$  can be determined using the experimental data. We use the parametrization of  $R_{\text{cont}}^{c\bar{c}}$  given in [19] (see Appendix). In Fig. 1, the imaginary part of  $g(\hat{m}_c, \hat{s})$ , Eq. (2.5), is plotted and compared with  $\text{Im } g(\hat{m}_c, \hat{s})$ , Eq. (3.3). Our numerical results are based on Eqs. (3.3) and (3.4), with the parameter  $\kappa_V$  chosen equal to 2.35.

#### IV. RATE AND FORWARD-BACKWARD ASYMMETRY

Neglecting nonperturbative corrections [20], the decay width as a function of the invariant mass of the lepton pair ( $q^2 \equiv m_{l^+l^-}^2$ ) is given by

$$\frac{d\Gamma}{d\hat{s}} = \frac{G_F^2 m_b^5}{192\pi^3} \frac{\alpha^2}{4\pi^2} |V_{tb} V_{ts}^*|^2 \lambda^{1/2}(1, \hat{s}, \hat{m}_s^2) \sqrt{1 - \frac{4\hat{m}_l^2}{\hat{s}}} \Delta, \quad (4.1)$$

where the factors  $\lambda$  and  $\Delta$  are defined by

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ac), \quad (4.2)$$

and

$$\begin{aligned} \Delta = & \left\{ \left( 12 \operatorname{Re}(c_7^{\text{eff}} c_9^{\text{eff}}) F_1(\hat{s}, \hat{m}_s^2) + \frac{4}{\hat{s}} |c_7^{\text{eff}}|^2 F_2(\hat{s}, \hat{m}_s^2) \right) \left( 1 + \frac{2\hat{m}_l^2}{\hat{s}} \right) \right. \\ & \left. + \left( |c_9^{\text{eff}}|^2 + |c_{10}|^2 \right) F_3(\hat{s}, \hat{m}_s^2, \hat{m}_l^2) + 6 \hat{m}_l^2 \left( |c_9^{\text{eff}}|^2 - |c_{10}|^2 \right) F_4(\hat{s}, \hat{m}_s^2) \right\}, \end{aligned} \quad (4.3)$$

with

$$F_1(\hat{s}, \hat{m}_s^2) = (1 - \hat{m}_s^2)^2 - \hat{s}(1 + \hat{m}_s^2), \quad (4.4)$$

$$F_2(\hat{s}, \hat{m}_s^2) = 2(1 + \hat{m}_s^2)(1 - \hat{m}_s^2)^2 - \hat{s}(1 + 14\hat{m}_s^2 + \hat{m}_s^4) - \hat{s}^2(1 + \hat{m}_s^2), \quad (4.5)$$

$$F_3(\hat{s}, \hat{m}_s^2, \hat{m}_l^2) = (1 - \hat{m}_s^2)^2 + \hat{s}(1 + \hat{m}_s^2) - 2\hat{s}^2 + \lambda(1, \hat{s}, \hat{m}_s^2) \frac{2\hat{m}_l^2}{\hat{s}}, \quad (4.6)$$

$$F_4(\hat{s}, \hat{m}_s^2) = 1 - \hat{s} + \hat{m}_s^2. \quad (4.7)$$

If the lepton mass  $m_l$  is neglected, we recover the results of Ali *et al.* [10, 11]. If the strange-quark mass  $m_s$  is also neglected, we obtain the results of Grinstein *et al.* [5] and Buras and Münz [4]. Finally, for  $m_l \neq 0$  but  $m_s = 0$ , we reproduce the result given by Hewett [13].

To complete the correspondence with previous results, we give here the forward-backward asymmetry of the lepton  $l^-$  in the  $l^+l^-$  centre-of-mass system:

$$A_{\text{FB}}(\hat{s}) = -3 \lambda^{1/2}(1, \hat{s}, \hat{m}_s^2) \sqrt{1 - \frac{4\hat{m}_l^2}{\hat{s}}} \frac{c_{10} \left[ \hat{s} \operatorname{Re} c_9^{\text{eff}} + 2 c_7^{\text{eff}} (1 + \hat{m}_s^2) \right]}{\Delta}. \quad (4.8)$$

This agrees with Ref. [10] when  $m_l$  is neglected.

## V. LEPTON-POLARIZATION

We now proceed to a discussion of the inclusive lepton polarization. We define three orthogonal unit vectors

$$\begin{aligned}\mathbf{e}_L &= \frac{\mathbf{p}_-}{|\mathbf{p}_-|} , \\ \mathbf{e}_N &= (\mathbf{p}_s \times \mathbf{p}_-)/|\mathbf{p}_s \times \mathbf{p}_-| , \\ \mathbf{e}_T &= \mathbf{e}_N \times \mathbf{e}_L ,\end{aligned}\tag{5.1}$$

where  $\mathbf{p}_-$  and  $\mathbf{p}_s$  are three-momenta of the  $l^-$  and the  $s$  quark, respectively, in the c.m. frame of the  $l^+l^-$  system. The differential decay rate of  $B \rightarrow X_s l^+l^-$  for any given spin direction  $\mathbf{n}$  of the lepton  $l^-$ , where  $\mathbf{n}$  is a unit vector in the  $l^-$  rest frame, may then be written as

$$\frac{d\Gamma(\mathbf{n})}{d\hat{s}} = \frac{1}{2} \left( \frac{d\Gamma}{d\hat{s}} \right)_{\text{unpol}} \left[ 1 + (P_L \mathbf{e}_L + P_T \mathbf{e}_T + P_N \mathbf{e}_N) \cdot \mathbf{n} \right], \tag{5.2}$$

where  $P_L$ ,  $P_T$ ,  $P_N$  are functions of  $\hat{s}$ , which give the longitudinal, transverse and normal components of polarization. The polarization component  $P_i$  ( $i = L, T, N$ ) is obtained by evaluating

$$P_i(\hat{s}) = \frac{d\Gamma(\mathbf{n} = \mathbf{e}_i)/d\hat{s} - d\Gamma(\mathbf{n} = -\mathbf{e}_i)/d\hat{s}}{d\Gamma(\mathbf{n} = \mathbf{e}_i)/d\hat{s} + d\Gamma(\mathbf{n} = -\mathbf{e}_i)/d\hat{s}}. \tag{5.3}$$

Our results for the polarization components  $P_L$ ,  $P_T$  and  $P_N$  are as follows

$$\begin{aligned}P_L(\hat{s}) &= \sqrt{1 - \frac{4\hat{m}_l^2}{\hat{s}}} \left[ 12 c_7^{\text{eff}} c_{10} \left( (1 - \hat{m}_s^2)^2 - \hat{s}(1 + \hat{m}_s^2) \right) \right. \\ &\quad \left. + 2 \text{Re}(c_9^{\text{eff}} c_{10}) \left( (1 - \hat{m}_s^2)^2 + \hat{s}(1 + \hat{m}_s^2) - 2\hat{s}^2 \right) \right] / \Delta ,\end{aligned}\tag{5.4}$$

$$P_T(\hat{s}) = \frac{3\pi\hat{m}_l}{2\sqrt{\hat{s}}} \lambda^{1/2}(1, \hat{s}, \hat{m}_s^2) \left[ c_7^{\text{eff}} c_{10} (1 - \hat{m}_s^2) - 4 \text{Re}(c_7^{\text{eff}} c_9^{\text{eff}}) (1 + \hat{m}_s^2) \right]$$

$$-\frac{4}{\hat{s}} |c_7^{\text{eff}}|^2 (1 - \hat{m}_s^2)^2 + \text{Re}(c_9^{\text{eff}} c_{10})(1 - \hat{m}_s^2) - |c_9^{\text{eff}}|^2 \hat{s} \big] / \Delta, \quad (5.5)$$

$$P_N(\hat{s}) = \frac{3\pi\hat{m}_l}{2\Delta} \text{Im}(c_9^{\text{eff}*} c_{10}) \sqrt{\hat{s}} \lambda^{1/2}(1, \hat{s}, \hat{m}_s^2) \sqrt{1 - \frac{4\hat{m}_l^2}{\hat{s}}}. \quad (5.6)$$

The expression for  $P_L$  agrees with that obtained by Hewett [13], when we set  $\hat{m}_s = 0$  in Eq. (5.4).

It should be noted that the polarization components  $P_L$ ,  $P_T$  and  $P_N$  involve different quadratic functions of the effective couplings  $c_7^{\text{eff}}$ ,  $c_9^{\text{eff}}$  and  $c_{10}$ , and therefore contain independent information. The component  $P_N$  is proportional to the absorptive part of the effective coupling  $c_9^{\text{eff}}$ , which is dominated by the charm-quark contribution (cf. Eq. (2.3)). It is obvious that the polarization is affected by any alteration in the relative magnitude and sign of  $c_7^{\text{eff}}$ ,  $c_9^{\text{eff}}$  and  $c_{10}$ , and thus can serve as a probe of possible new interactions transcending the standard model.

The polarization components  $P_L$ ,  $P_T$  and  $P_N$  are plotted in Figs. 2–5. In the case of the  $\mu^+\mu^-$  channel, the only significant component is  $P_L$ , which has a large negative value over most of the  $\hat{s}$  domain, with an average value  $\langle P_L \rangle_\mu = -0.77$ . By contrast, all three components are sizeable in the  $\tau^+\tau^-$  channel, with average values  $\langle P_L \rangle_\tau = -0.37$ ,  $\langle P_T \rangle_\tau = -0.63$ ,  $\langle P_N \rangle_\tau = 0.03$  (0.02) for  $\kappa_V = 2.35$  (1). Notice that the  $T$ -odd component  $P_N$ , though small, is considerably larger than the corresponding normal polarization of leptons in  $K_L \rightarrow \pi^+\mu^-\bar{\nu}$  or  $K^+ \rightarrow \pi^+\mu^+\mu^-$ , which requires a final state Coulomb interaction of the lepton with the other charged particles, and is typically of order  $\alpha(m_\mu/m_K) \sim 10^{-3}$  [21].

The inclusive branching ratios are predicted to be  $\text{Br}(B \rightarrow X_s \mu^+\mu^-) = 6.7 \times 10^{-6}$ ,  $\text{Br}(B \rightarrow X_s \tau^+\tau^-) = 2.5 \times 10^{-7}$ . The lower rate of the  $\tau^+\tau^-$  channel may be offset by the fact that the decay of the  $\tau$  acts as a self-analyser of the  $\tau$  polarization. Assuming (as in Ref. [13]) a total of  $5 \times 10^8$   $B\bar{B}$  decays, one can expect to observe  $\sim 100$  identified  $B \rightarrow X_s \tau^+\tau^-$  events, permitting a test of the predicted polarizations  $\langle P_L \rangle = -0.37$ , and  $\langle P_T \rangle = -0.63$  with good accuracy. We shall discuss in a more



detailed paper the dependence of the lepton polarization on the production angle  $\theta$ , the spin correlation of the  $l^+l^-$  pair, the influence of nonperturbative effects (quark-binding corrections), as well as lepton-spin effects in exclusive channels.

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## APPENDIX: INPUT PARAMETERS

$$\begin{aligned}
m_b &= 4.8 \text{ GeV}, \quad m_c = 1.4 \text{ GeV}, \quad m_s = 0.2 \text{ GeV}, \quad m_t = 176 \text{ GeV} , \\
m_\mu &= 0.106 \text{ GeV}, \quad m_\tau = 1.777 \text{ GeV}, \quad M_W = 80.2 \text{ GeV}, \quad \mu = m_b , \\
V_{tb} &= 1, \quad V_{ts}^* = -V_{cb}, \quad \text{Br}(B \rightarrow X_c l \bar{\nu}_l) = 10.4\% , \\
\Lambda_{\text{QCD}} &= 225 \text{ MeV}, \quad \alpha = 1/129, \quad \sin^2\theta_W = 0.23 .^3
\end{aligned} \tag{A1}$$

$$R_{\text{cont}}^{c\bar{c}}(\hat{s}) = \begin{cases} 0 & \text{for } 0 \leq \hat{s} \leq 0.60 , \\ -6.80 + 11.33\hat{s} & \text{for } 0.60 \leq \hat{s} \leq 0.69 , \\ 1.02 & \text{for } 0.69 \leq \hat{s} \leq 1 . \end{cases} \tag{A2}$$

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<sup>3</sup>We use  $\alpha_s(\mu)$  that is given by the formula (4.12) of Ref. [4].

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## FIGURE CAPTIONS

**Figure 1** The imaginary part  $\text{Im } g(\hat{m}_c, \hat{s})$  as a function of the invariant mass of the lepton pair. The dashed line represents the theoretical result, neglecting long-distance effects, and the solid curve shows the imaginary part using  $R_{\text{had}}^{c\bar{c}}(\hat{s})$ , as described in the text.

**Figure 2** The longitudinal polarization  $P_L$  for the  $\mu^-$  including the  $c\bar{c}$  resonances ( $\kappa_V = 2.35$ ).

**Figure 3** The longitudinal polarization  $P_L$  for the tau lepton including  $c\bar{c}$  resonances.

**Figure 4** The transverse polarization  $P_T$  (in the decay plane) of the  $\tau^-$  for  $\hat{s} \geq 4\hat{m}_\tau^2$ .

**Figure 5** The normal polarization  $P_N$ , i.e. normal to the decay plane, in the  $\tau^+\tau^-$  channel including  $c\bar{c}$  intermediate states. The solid line corresponds to  $\kappa_V = 2.35$ , the dashed one is  $\kappa_V = 1$ .

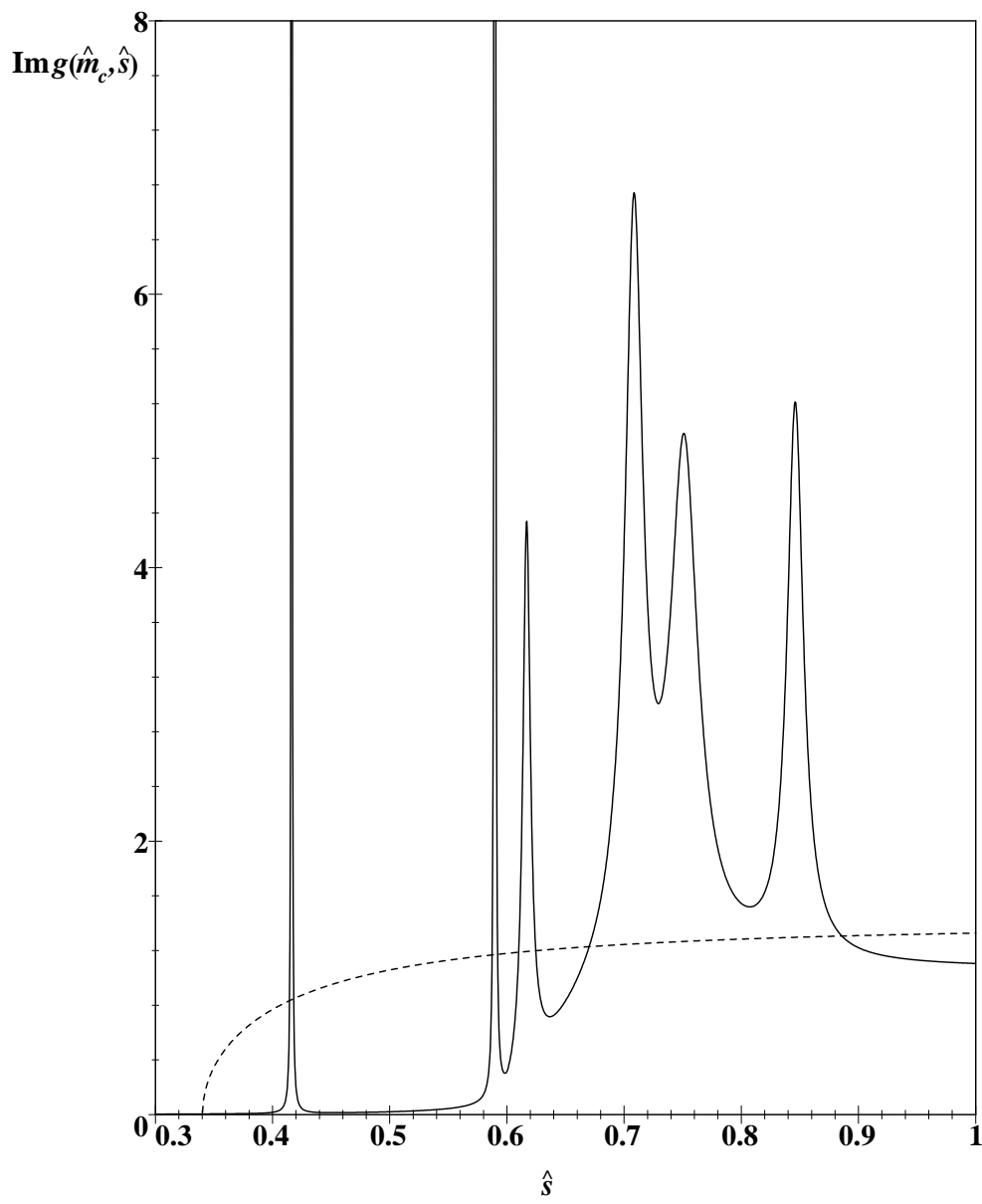


FIG. 1:

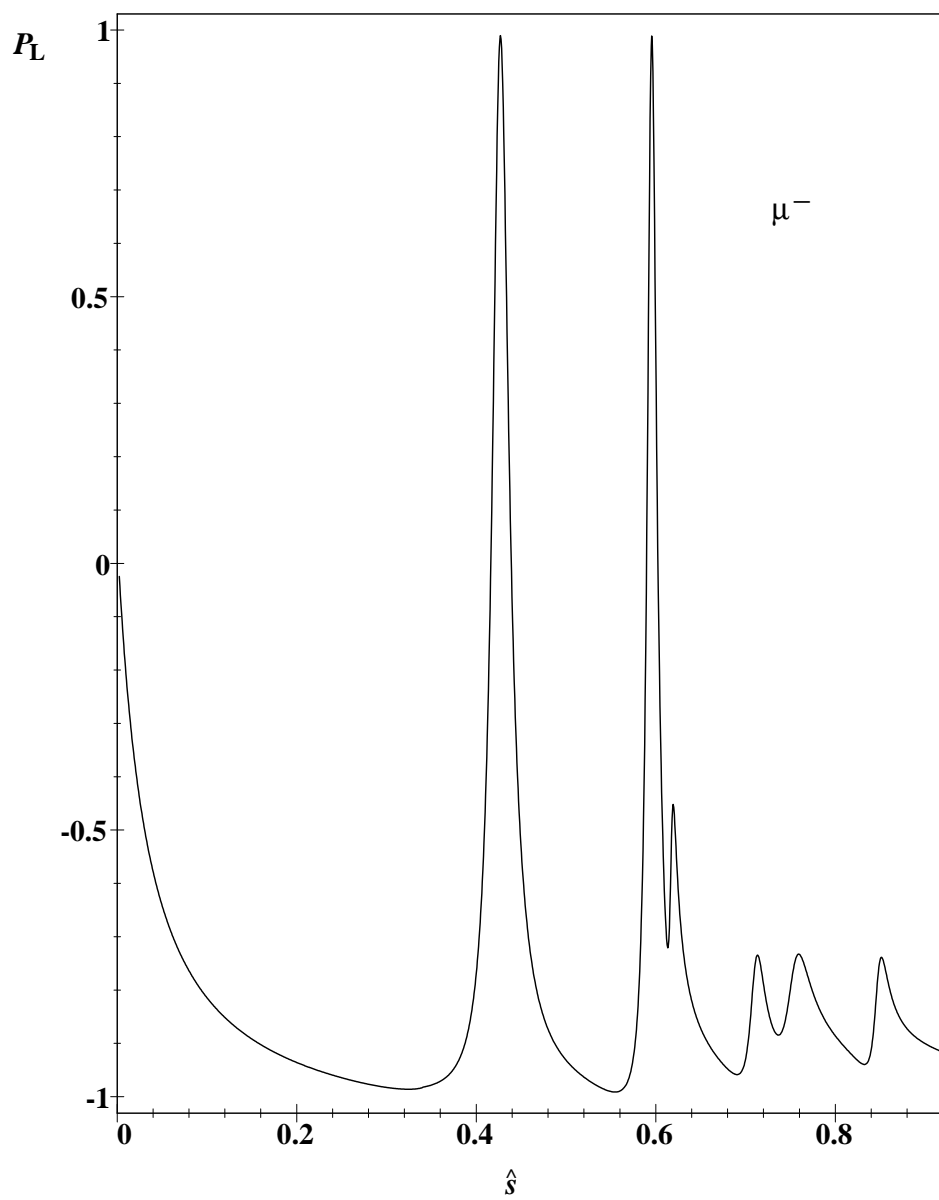


FIG. 2:

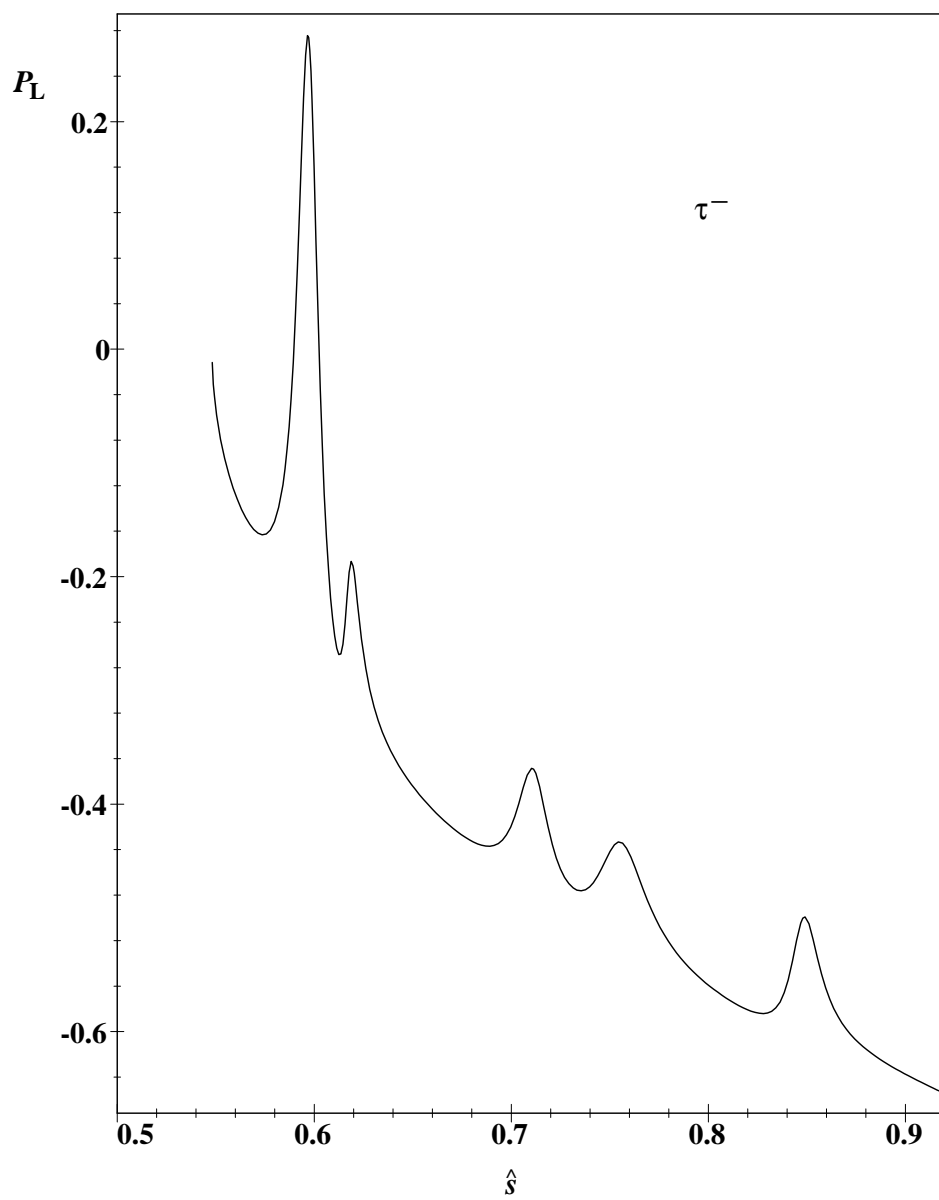


FIG. 3:

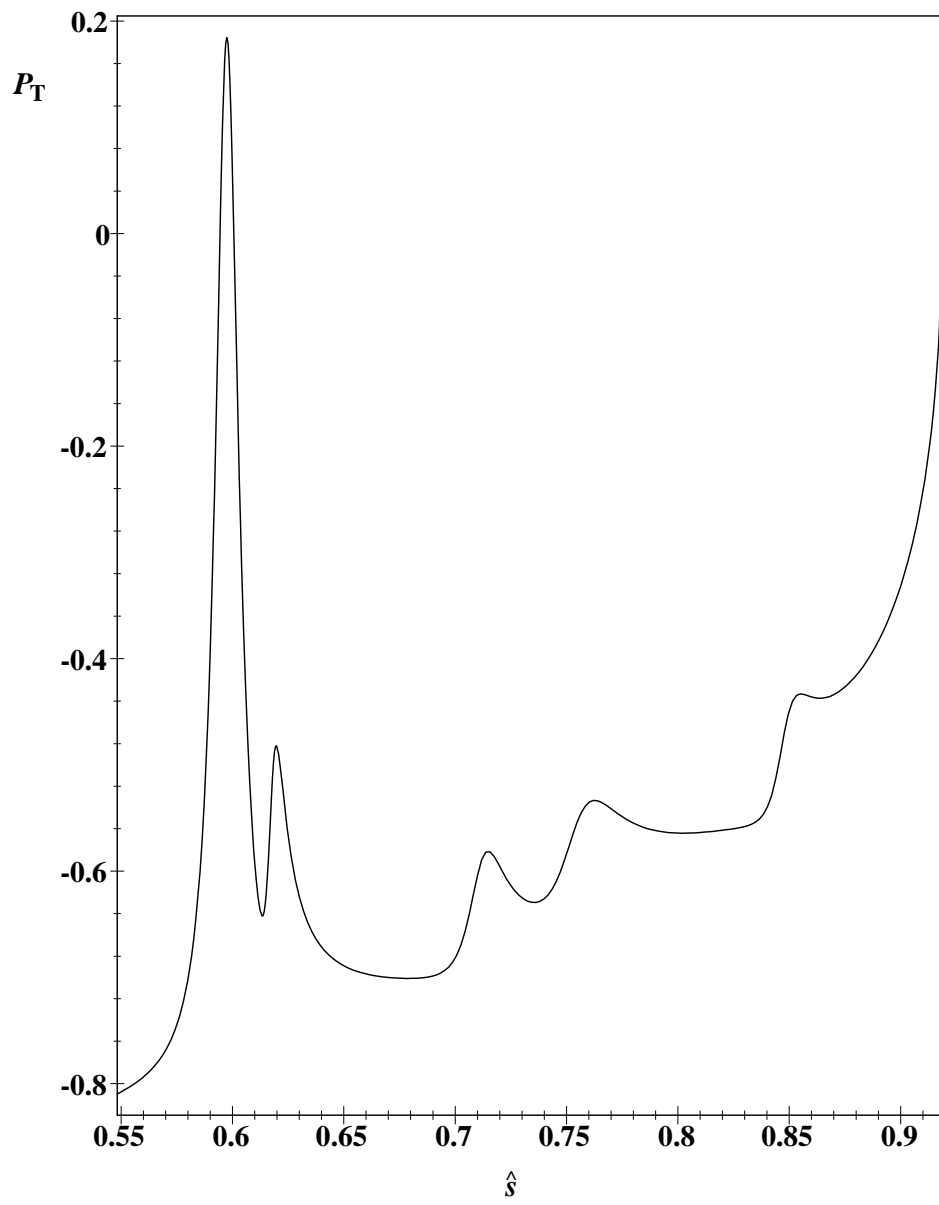


FIG. 4:



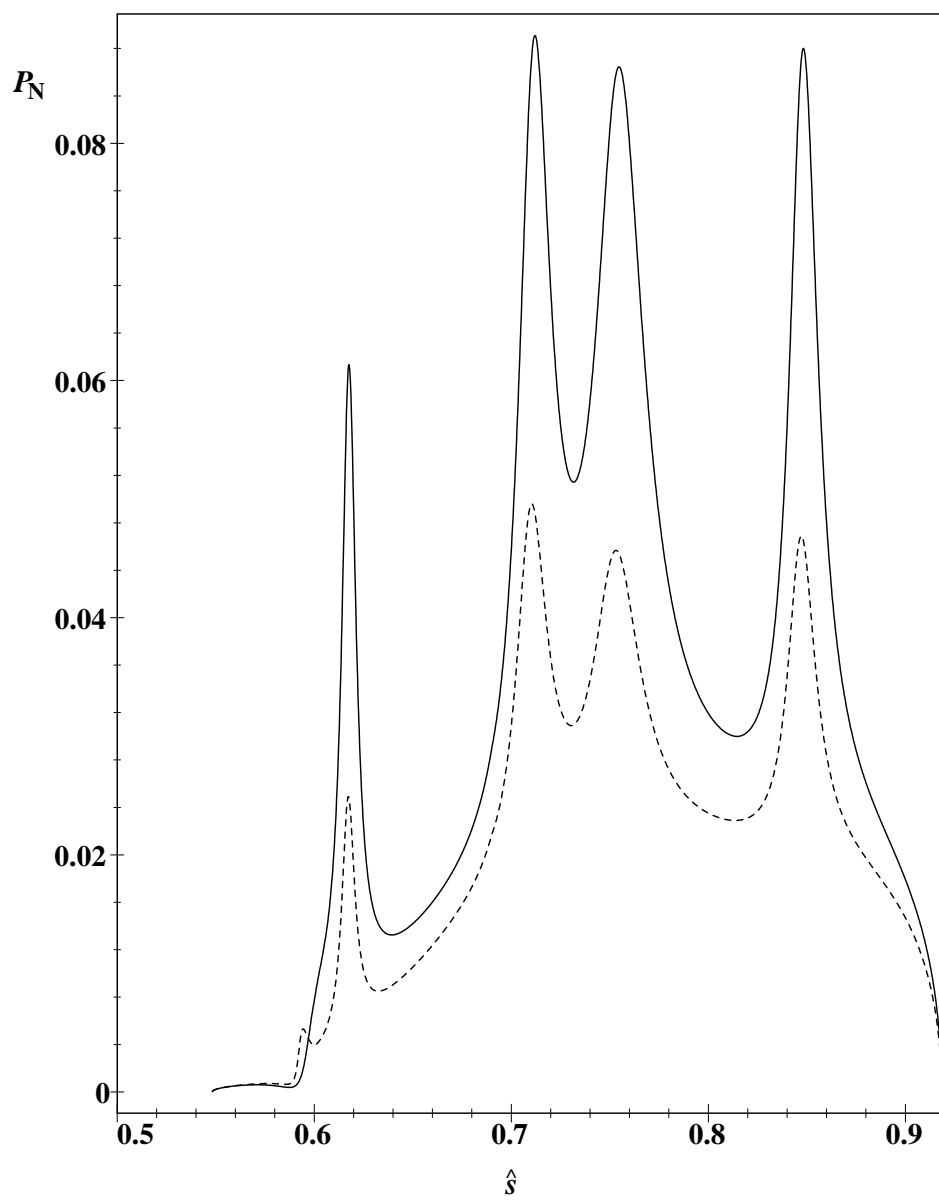


FIG. 5: